

The University of Chicago

FACTORS IN THE GROWTH OF GIRLS

A PART OF A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE
SOCIAL SCIENCES IN CANDIDACY FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF EDUCATION

1939

By

FRANCES A. MULLEN

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CHILD DEVELOPMENT, Vol. XI, No. 1 (March, 1940)

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FACTORS IN THE GROWTH OF GIRLS¹

FRANCES A. MULLEN²

The intercorrelations of seventeen physical measurements of girls of various age groups have been analyzed to determine: 1) the portion of the variance at each age which may be ascribed to a general growth factor; 2) the group and special factors which may be necessary to account for the residual correlations at each age; 3) whether one estimate of the general and group factors may be used at all ages, in the study of the growth of the factors, and 4) the relationship of such factors, both general and group, to the age at which puberty occurs.

The intercorrelations have been analyzed by the bi-factor technique developed by Holzinger (4). This method of factor analysis assumes a general factor to account for the positive character of the correlational matrix, and independent group factors to account for higher correlations within a restricted group of measurements than would be produced by the general factor alone. To the general factor is ascribed as much of the variance of each individual measure as is consonant with the magnitude of the correlation coefficients and with the postulated group factors. The residual correlations are then accounted for by group factors which are the general factors common to a restricted group of residuals, and by special factors for each measurement.

The analysis presented in this paper will show that it is possible to obtain an excellent fit to the present correlation tables by such an assumption of general, group, and special factors, all independent of each other. It would have been possible, however, to have postulated group factors without a general factor, even though the correlations are all positive. In such a case the group factors must either overlap, as they do in most multiple factor analyses, where each factor has loadings in practically all of the measurements, or the group factors must themselves be correlated, not independent. The choice between these methods of factoring the observed correlations must be made not on mathematical grounds but in view of such questions as: Which method gives the most understandable and parsimonious description of the factors? Which analysis gives results of most use to the geneticist or student of child development in his effort to understand the underlying phenomena?

The subjects of the study are girls enrolled in the Laboratory Schools of the University of Chicago. The measurements, made by physicians on the staff of the schools during the years 1927-37, are as follows: height, arm span, length of right forearm, length of right lower leg, sitting height, weight, bi-iliac diameter, bi-trochanteric diameter, chest girth, chest width, chest depth, shoulder width, head length, head width, head height, lung capacity, and right hand squeeze. The technique used in making and recording each measurement is described by Bolmeier (1). The measurements were made as nearly as possible on the date of the child's birthday. For the records used in this study the average date of

¹The writer wishes to acknowledge her indebtedness to Dr. Karl J. Holzinger, under whose direction this study was carried out, and to Dr. Frank N. Freeman, who made available the records of the Laboratory Schools of the University of Chicago.

²From the Bureau of Child Study, Board of Education, Chicago.

examination was 2.005 days after the birthday, with a mean deviation of 6.24 days. Since each age group is treated separately, age is well controlled and need not be allowed for statistically.

The study is limited to girls of the ages of seven, nine, eleven, thirteen, fifteen, and seventeen. All records in the files for girls of these ages were used in this study if they were complete in the seventeen selected measurements, with the exception of seven records discarded because of inconsistencies.

The 136 possible intercorrelations of the seventeen measurements at each age level were computed by the product-moment technique. In analyzing the resulting correlational matrices, two clusters of variables became evident. The five variables: height, arm span, length of right forearm, length of right lower leg, and sitting height have at all ages higher intra-group than extra-group correlations. Similarly, the seven variables: weight, the two hip measures, the three chest measures, and shoulder width, form a second cluster. The identification of these clusters was made by the use of the B-coefficient (4, p. 23) and verified by the goodness of fit of the resulting factor patterns. Sitting height, however, proved on further study to be related to the first group largely through its high correlation with standing height and to have insignificant factor loadings with the first group factor. Similarly shoulder width is the most doubtful member of the second group. No grouping could be set up among the remaining five variables.

At most age levels, four correlations appeared to be significantly higher than could be accounted for by the general and group factors here postulated. In the final calculations of the factor loadings, the results of which are shown in Tables 1 to 6, allowance was made for these overlaps.

The entries in these tables may be interpreted in several ways. Each factor loading is the correlation of the given variable with the hypothetical factor. Each loading is also the square root of the proportion of the total variance of a given measure which may be ascribed to that factor. Finally, the correlation between any two variables should be reproduced by the sum of the products of the factor loadings which the two variables have in common.

The various measures correlate much more highly with the general factor than they do with the two group factors. For the last five measures, however, and for some of the second group of measures, a large part of the variance cannot be ascribed to general or group factors, or to doublets, but must be accounted for by factors unique to each measure.

To check the accuracy with which the factor patterns fit the observed correlations, the differences between the theoretical correlations (computed from the sum of the products of the loadings of such factors as the two variables have in common) and the observed correlations have been computed. It should not be expected that these residuals will all be exactly zero, but they should distribute around zero. The means and variabilities of the residuals for each age are shown in Table 7. It will be noted that .6745 times the standard deviation of each of these distributions is less than the probable error of a zero correlation for a population of the size of each of these age groups.

The probable error of a zero correlation has been suggested as a

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TABLE 1

FACTOR LOADINGS FROM THE CORRELATIONS BETWEEN SEVENTEEN
PHYSICAL MEASUREMENTS OF 189 SEVEN-YEAR-OLD GIRLS

Variables	α	β	γ	D ₁	D ₂	D ₃	D ₄	ω_i
Height	.878	.372	-	-	-	-	.000	.301
Arm span	.797	.488	-	.000	-	-	-	.356
Forearm	.756	.568	-	-	-	-	-	.326
Lower leg	.823	.440	-	-	-	-	-	.359
Sitting height	.832	.000	-	-	-	-	.000	.555
Weight	.834	-	.445	-	-	-	-	.326
Bi-iliac diameter	.677	-	.451	-	-	-	-	.581
Bi-trochanteric "	.744	-	.488	-	-	-	-	.456
Chest girth	.682	-	.466	-	-	-	-	.564
Chest width	.590	-	.432	-	-	-	-	.682
Chest depth	.527	-	.336	-	-	-	-	.780
Shoulder width	.703	-	.200 ^a	.000	-	-	-	.682
Head length	.398	-	-	-	.444	-	-	.803
Head width	.352	-	-	-	-	.270	-	.896
Head height	.323	-	-	-	.444	.270	-	.791
Hand squeeze	.299	-	-	-	-	-	-	.954
Lung capacity	.397	-	-	-	-	-	-	.918
Total variance	7.273	.893	1.196	.000	.394	.146	.000	7.096
Per cent variance	42.8	5.4	7.0	.0	2.3	.9	.0	41.7

^a Insignificant.

TABLE 2

FACTOR LOADINGS FROM THE CORRELATIONS BETWEEN SEVENTEEN
PHYSICAL MEASUREMENTS OF 212 NINE-YEAR-OLD GIRLS

Variables	α	β	γ	D ₁	D ₂	D ₃	D ₄	ω_i
Height	.840	.424	-	-	-	-	.210	.266
Arm span	.792	.510	-	.179	-	-	-	.284
Length of forearm	.796	.543	-	-	-	-	-	.268
Length of lower leg	.868	.318	-	-	-	-	-	.382
Sitting height	.773	.049 ^a	-	-	-	-	.210	.597
Weight	.768	-	.542	-	-	-	-	.341
Bi-iliac diameter	.668	-	.487	-	-	-	-	.563
Bi-trochanteric "	.611	-	.642	-	-	-	-	.464
Chest girth	.582	-	.598	-	-	-	-	.551
Chest width	.544	-	.521	-	-	-	-	.658
Chest depth	.465	-	.565	-	-	-	-	.682
Shoulder width	.626	-	.346	.179	-	-	-	.676
Head length	.482	-	-	-	.438	-	-	.759
Head width	.248	-	-	-	-	.300	-	.921
Head height	.291	-	-	-	.438	.300	-	.796
Hand squeeze	.544	-	-	-	-	-	-	.839
Lung capacity	.514	-	-	-	-	-	-	.858
Total variance	6.908	.838	2.011	.064	.384	.180	.088	6.531
Per cent variance	40.6	4.9	11.8	.4	2.3	1.1	.5	38.4

^a Insignificant.

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TABLE 3

FACTOR LOADINGS FROM THE CORRELATIONS BETWEEN SEVENTEEN
PHYSICAL MEASUREMENTS OF 230 ELEVEN-YEAR-OLD GIRLS

Variables	α	β	γ	D ₁	D ₂	D ₃	D ₄	ω_i
Height	.888	.344	-	-	-	-	.207	.224
Arm span	.863	.403	-	.017	-	-	-	.304
Length of forearm	.857	.472	-	-	-	-	-	.207
Length of lower leg	.871	.338	-	-	-	-	-	.356
Sitting height	.833	.000	-	-	-	-	.207	.513
Weight	.784	-	.536	-	-	-	-	.313
Bi-iliac diameter	.735	-	.495	-	-	-	-	.464
Bi-trochanteric "	.750	-	.529	-	-	-	-	.397
Chest girth	.755	-	.467	-	-	-	-	.460
Chest width	.628	-	.595	-	-	-	-	.502
Chest depth	.551	-	.499	-	-	-	-	.669
Shoulder width	.680	-	.255	.017	-	-	-	.687
Head length	.401	-	-	-	.381	-	-	.833
Head width	.237	-	-	-	-	.363	-	.901
Head height	.378	-	-	-	.381	.363	-	.762
Hand squeeze	.599	-	-	-	-	-	-	.801
Lung capacity	.577	-	-	-	-	-	-	.817
Total variance	8.220	.618	1.698	.006	.290	.264	.086	5.825
Per cent variance	48.4	3.6	10.0	.0	1.7	1.5	.5	34.3

TABLE 4

FACTOR LOADINGS FROM THE CORRELATIONS BETWEEN SEVENTEEN
PHYSICAL MEASUREMENTS OF 219 THIRTEEN-YEAR-OLD GIRLS

Variables	α	β	γ	D ₁	D ₂	D ₃	D ₄	ω_i
Height	.769	.552	-	-	-	-	.276	.167
Arm span	.712	.611	-	.000	-	-	-	.348
Length of forearm	.661	.711	-	-	-	-	-	.241
Length of lower leg	.675	.541	-	-	-	-	-	.502
Sitting height	.742	.136 ^a	-	-	-	-	.276	.596
Weight	.752	-	.580	-	-	-	-	.313
Bi-iliac diameter	.661	-	.355	-	-	-	-	.661
Bi-trochanteric "	.738	-	.416	-	-	-	-	.531
Chest girth	.500	-	.625	-	-	-	-	.599
Chest width	.603	-	.523	-	-	-	-	.602
Chest depth	.429	-	.401	-	-	-	-	.809
Shoulder width	.699	-	.196 ^a	-	-	-	-	.688
Head length	.439	-	-	-	.000	-	-	.898
Head width	.286	-	-	-	-	.497	-	.819
Head height	.470	-	-	-	.000	.497	-	.729
Hand squeeze	.544	-	-	-	-	-	-	.839
Lung capacity	.629	-	-	-	-	-	-	.777
Total variance	6.554	1.495	1.499	.000	.000	.494	.152	6.798
Per cent variance	38.6	8.8	8.8	.0	.0	2.9	.9	40.0

^a Insignificant.

TABLE 5

FACTOR LOADINGS FROM THE CORRELATIONS BETWEEN SEVENTEEN
PHYSICAL MEASUREMENTS OF 305 FIFTEEN-YEAR-OLD GIRLS

Variables	α	β	σ	D ₁	D ₂	D ₃	D ₄	ω_i
Height	.691	.614	-	-	-	-	.354	.142
Arm span	.591	.740	-	.251	-	-	-	.200
Length of forearm	.581	.704	-	-	-	-	-	.409
Length of lower leg	.598	.652	-	-	-	-	-	.466
Sitting height	.674	.116 ^a	-	-	-	-	.354	.638
Weight	.694	-	.623	-	-	-	-	.361
Bi-iliac diameter	.551	-	.339	-	-	-	-	.762
Bi-trochanteric "	.611	-	.560	-	-	-	-	.559
Chest girth	.562	-	.453	-	-	-	-	.692
Chest width	.596	-	.473	-	-	-	-	.649
Chest depth	.407	-	.362	-	-	-	-	.838
Shoulder width	.579	-	.089 ^a	.500	-	-	-	.638
Head length	.330	-	-	-	.327	-	-	.886
Head width	.383	-	-	-	-	.292	-	.876
Head height	.297	-	-	-	.327	.292	-	.848
Hand squeeze	.496	-	-	-	-	-	-	.868
Lung capacity	.695	-	-	-	-	-	-	.719
Total variance	5.380	1.859	1.385	.313	.214	.171	.251	7.427
Per cent variance	31.6	10.9	8.1	1.8	1.3	1.0	1.5	43.7

^aInsignificant.

TABLE 6

FACTOR LOADINGS FROM THE CORRELATIONS BETWEEN SEVENTEEN
PHYSICAL MEASUREMENTS OF 165 SEVENTEEN-YEAR-OLD GIRLS

Variables	α	β	σ	D ₁	D ₂	D ₃	D ₄	ω_i
Height	.620	.624	-	-	-	-	.365	.305
Arm span	.546	.756	-	.276	-	-	-	.233
Length of forearm	.501	.783	-	-	-	-	-	.369
Length of lower leg	.572	.713	-	-	-	-	-	.405
Sitting height	.562	.142 ^a	-	-	-	-	.365	.729
Weight	.764	-	.531	-	-	-	-	.366
Bi-iliac diameter	.466	-	.405	-	-	-	-	.787
Bi-trochanteric "	.606	-	.500	-	-	-	-	.619
Chest girth	.609	-	.283	-	-	-	-	.741
Chest width	.670	-	.446	-	-	-	-	.593
Chest depth	.355	-	.233	-	-	-	-	.906
Shoulder width	.706	-	.170 ^a	.276	-	-	-	.630
Head length	.410	-	-	-	.395	-	-	.822
Head width	.210	-	-	-	-	.000	-	.978
Head height	.238	-	-	-	.395	.000	-	.887
Hand squeeze	.322	-	-	-	-	-	-	.947
Lung capacity	.645	-	-	-	-	-	-	.764
Total variance	4.977	2.103	1.058	.152	.312	.000	.266	8.132
Per cent variance	29.3	12.4	6.2	.9	1.8	.0	1.6	47.8

^aInsignificant.

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TABLE 7

MEANS AND VARIABILITIES OF THE DISTRIBUTIONS OF
RESIDUAL CORRELATIONS FOR EACH AGE GROUP

Age	Mean	Standard Deviation	.6745 S.D.	P.E. $r=0$
7	+.001	.044	.030	.049
9	+.003	.045	.031	.046
11	-.002	.048	.032	.044
13	+.001	.052	.035	.045
15	+.001	.056	.038	.038
17	-.001	.056	.038	.052

criterion of the insignificance of the residuals (4, p. 57), but is possibly too low a criterion, for the factor loadings are not raw correlations but are derived from them and might be expected to be subject to chance errors greater than those of corresponding raw correlations. The factor patterns given in Tables 1 to 6 appear to reproduce the tables of correlation with a degree of accuracy slightly greater than might be expected in view of the probable errors of the indices involved.

The permissive character of any factor analysis from the statistical point of view has been granted. In support of the scheme of factors here presented, it can be said that it fits the observed data, i.e., the tables of correlations, and that it has the advantage of reducing a large set of facts to a parsimonious description. The hypothesis of a general factor plus two independent group factors seems to present a neater and more parsimonious and interpretable picture of the observed data than do the factor systems derived from similar sets of data by workers (2, 5, 6, 7) who have used the multiple-factor methods and who present overlapping group factors each of which has significant weightings in such a variety of measurements as to make its interpretation very difficult.

Wright analyzed the intercorrelations of various skeletal measurements of several populations of rabbits and fowl by still another statistical technique, that of path coefficients. He found that the influence of the general size factor predominates, but that the residuals indicated the existence of group factors for the head, the forelimbs and hind limbs collectively, the hind limbs separately, and for the wings in fowl (10, p. 619).

THE CONSISTENCY OF THE FACTOR PATTERNS

The general factor loadings of the linear variables are on the whole higher than those for the cross-sectional measurements at all ages except seventeen. In other words the linear measurements are more highly correlated with the general factor at the early ages, and especially at age eleven, but the trend begins to reverse at this point, and by the age of seventeen the cross-sectional measures correlate more highly with the general factor. If it is assumed that this general factor is a mathematical statement of some underlying genetic or environmental cause (or causes operating as a unit), which determines the general size of the various parts, it apparently operates differently after the age of eleven than before it.

This change in the factor loadings from one age to another raises pertinent questions which the data at hand cannot answer. Are the factors indicated in one pattern the same as those indicated in another? Are the factors unitary or compound? Do the underlying factors determining growth change from age to age, or are the factors really the same, but our measurements of these factors obscured in varying degrees at varying ages, by the fact that two or more factors are operating in the same direction at one period but not at another, or by other difficulties?

The fact that the factors are as much alike as they are from one age group to another (that the same group factors emerge in each pattern and that the loadings are as consistent as they are) is some indication that there are real underlying factors of which our statistically determined factors are at least useful approximations, and that these underlying factors are constant. The problems arising in the task of estimating the size factors will throw additional light on this question of the consistency of the factors.

THE GROWTH OF THE FACTORS

As a basis for the calculation of the regression equations from which the factors might be estimated for individuals or groups of persons, three composite variables were set up, all the measurements which correlate with the factor β being combined into a composite variable B, all those which correlate with the factor σ being combined into a composite variable C, and the remaining five measurements into a composite variable D. The general factor and the group factor β were then computed from a seven-rowed correlational matrix consisting of the five variables of the β group in their original form and the composite variables C and D, while the group factor σ was computed from a 9-rowed correlational matrix consisting of the seven variables of the σ group in their original form and the composite variables B and D. Harman (3) has shown that there is small loss of reliability in such a procedure.

Each factor pattern, of course, gives a different equation of estimate, applicable directly only to the age group from which it was obtained. If any growth of factors is to be traced it is necessary to select one equation and apply it to girls of all age groups.

The equations are first determined in a form applicable to the standard scores in the various measures. In the process of reducing them to a form applicable to raw scores in the original measurements, the standard deviation of the measurements is used. Since the equation finally selected was to be applied not to any one age group but to all, new coefficients of standard deviation were calculated for each measurement by throwing the distributions for all age groups together and computing the standard deviation of the distribution of each measurement among the entire population studied.

The six equations for each factor were applied first to sample groups of girls of certain age groups to determine the correlation between estimates made by the different equations, and then to the average of each age group to determine whether they revealed similar trends of growth of the factors.

Estimates of α by three different equations gave correlations ranging

from .91 to .99+ for various age groups, and the growth curves were so similar as to indicate that it made little difference which equation was used.

The estimates of the group factors, as might be expected, appear to be considerably less stable than those of the general factor. The six different equations for the estimation of β are similar in some respects but the differences in the weightings of the measures within the β group are noticeable. Estimates for a sample of 34 girls of the nine-year-old group by two of these equations gave a correlation of $.748 \pm .048$. It is therefore much less certain whether it is justifiable to use any one of the six equations for estimates of β at all ages. However, if any growth is to be traced, some equation must be selected.

The mean values of the estimates of β for each age group as calculated by each of the six equations are shown in Table 8 and graphically in Figure 1, together with the average of the estimates at each age.

Estimates of β based on the equation for the eleven-year-old group show the greatest deviation from the typical form of the curves of mean estimate. The equation for β_9 gives results most closely agreeing with the average trend, and has a higher reliability ($R = .906$) than has the equation for β_7 ($R = .782$) which gives results almost equally close to the average trend. β_9 will therefore be used hereafter in this report for estimating the factor for individuals or groups. It must be admitted, however, that the differences between the different regression equations for β , the lack of high correlation between the estimates by two of them, and the differences in these growth curves all throw doubt on the advisability of attempting to use one equation of estimate at all ages.

The regression equations for the estimation of σ show even less reliability than those of β , as measured by the multiple correlation coefficients of the regression equations. The correlation between estimates of σ by the equations for σ_9 and for σ_{15} for a sample of 34 girls of age nine is, however, practically the same, $.758 \pm .048$, as the corresponding correlation for β estimates.

The mean value of the estimates of σ for each age group as calculated by each of the six equations is shown in Table 9 and in Figure 2, where the average value of the six estimates at each age is also shown for comparison.

The curve produced by the use of the equation for σ_{11} deviates most widely from the typical form of these curves. The other five are fairly similar in shape. The equation for σ_{15} most closely approximates the average curve of growth for these estimates of the factor σ , and has been selected for future use in estimating the factor.

TABLE 8
MEAN ESTIMATES OF β FOR EACH AGE GROUP,
ACCORDING TO SIX DIFFERENT EQUATIONS

Age	β_7	β_9	β_{11}	β_{13}	β_{15}	β_{17}	Average
7	20.00	20.00	20.00	20.00	20.00	20.00	20.00
9	21.50	21.01	21.20	21.06	21.43	21.59	21.30
11	22.54	21.61	21.65	21.69	22.54	22.89	22.15
13	23.16	22.20	21.62	22.06	23.40	23.80	22.71
15	22.87	22.06	20.97	21.79	23.33	23.80	22.47
17	22.65	21.91	20.65	21.64	23.14	23.58	22.26

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TABLE 9

MEAN ESTIMATES OF $\bar{r}$ FOR EACH AGE GROUP,
ACCORDING TO SIX DIFFERENT EQUATIONS

Age	$\bar{r}_7$	$\bar{r}_9$	$\bar{r}_{11}$	$\bar{r}_{13}$	$\bar{r}_{15}$	$\bar{r}_{17}$	Average
7	20.00	20.00	20.00	20.00	20.00	20.00	20.00
9	20.22	20.10	19.66	20.22	20.11	19.96	20.04
11	20.93	20.58	19.57	20.79	20.57	20.33	20.46
13	22.06	21.56	20.47	21.84	21.65	21.33	21.48
15	22.91	22.20	21.12	22.35	22.13	21.77	22.08
17	22.97	22.54	21.52	22.33	22.12	21.74	22.20

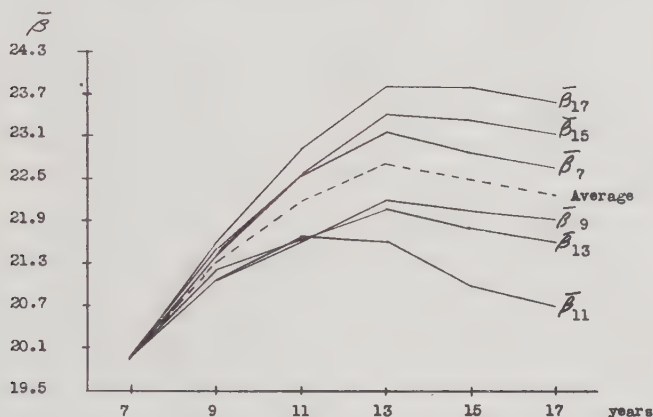


Fig. 1. Mean estimates of β for each age level according to six different regression equations.

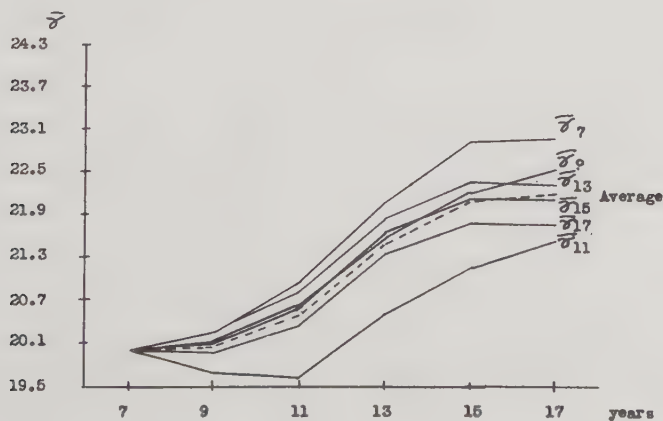


Fig. 2. Mean estimates of r for each age level according to six different regression equations.

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A comparison of the curves of Figure 1 with those for Figure 2 indicates typical differences between the form of the growth curves for the two group factors. The factor β shows rapid growth from the age of seven to that of eleven with a leveling off or decrease at thirteen. The σ factor characteristically shows very little growth from seven to nine, slight increases from nine to eleven, rapid growth from eleven to fifteen, with a leveling off at fifteen.

GROWTH OF THE FACTORS IN RELATION TO THE ADVENT OF THE MENARCHE

Since the advent of the menarche has been frequently shown to be closely associated with changes in the rate of physical growth, the girls of the present study, for whom the necessary data were available, have been subdivided into four sections according to their age at the menarche. Mean growth curves for each factor have been worked out for each maturity section. Section I includes girls for whom the menarche appeared during the eleventh year of age or earlier; Section II, girls for whom the menarche appeared during the twelfth year of age; Section III, thirteenth; and Section IV, girls for whom the menarche appeared after the fourteenth birthday.

The mean estimates for each maturity section at each age for both the general and group factors are shown in Table 10 and in Figures 3, 4, and 5. Since the evidence which has been presented suggests that estimates of the general factor may be made with high reliability by equations derived from any age group, the equation $\bar{a}_{9b}$ (derived for the nine-year-old girls with the linear variables in their original form and the others in two composites) has been arbitrarily selected for the estimation of α . The estimates of the two group factors have been made by equations $\bar{\beta}_9$ and $\bar{\tau}_{15}$, but as has been noted the estimates of the group factors have low multiple correlations and the results obtained here for the group factors must be considered as suggestive only.

The growth curves for the general factor present a consistent pattern. The earliest maturing section exceeds the other three at age seven, noticeably increases its superiority at age eleven, and has largely lost its advantage by the age of fifteen.

The two middle sections have noticeably decreased the differences between themselves and Section I at age 13, but the section latest in maturing is still considerably behind the other sections at that age.

The growth of the general factor, therefore, appears to be definitely and positively related to the stage of sexual maturity reached by the girls. The mean growth of a section of the girls in the general factor seems to be accelerated during the year of or just preceding the menarche, and retarded thereafter, the differences between the groups being negligible at the age of 17.

A very different picture is presented for the growth of the group factor β . Differences between the sections are slight up to the age of 11. Thereafter the curves fan out in inverse order, Section IV having consistently highest place and Section I the lowest. In noting this superiority of Section IV at the later ages, it must be remembered that high estimates of β might be obtained from the use of the regression equation through large mean values of the height and limb measures which

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TABLE 10

MEAN ESTIMATES OF THREE FACTORS IN THE PHYSICAL MEASUREMENTS
OF GIRLS OF VARIOUS MATURITY SECTIONS

Age	Mean Estimates for All Girls in This Study	Mean Estimates for			
		Section I (Menarche during 11th Year or Earlier)	Section II (Menarche during 12th Year)	Section III (Menarche during 13th Year)	Section IV (Menarche during 14th Year or Later)
General Factor, $\bar{a}_{9b}$					
7	20.00	20.40	19.36	19.27	19.25
9	22.44	22.90	22.39	21.92	21.78
11	24.99	26.59	24.93	24.46	23.64
13	27.39	27.99	27.74	27.14	25.96
15	28.57	28.55	28.78	28.52	28.35
17	28.92	28.90	28.90	29.04	28.80
Group Factor, $\bar{B}_9$					
7	20.00	20.42	20.42	20.56	20.22
9	21.01	21.16	21.01	21.21	20.84
11	21.61	21.62	21.79	21.01	21.50
13	22.20	21.65	22.22	22.53	22.63
15	22.06	21.27	21.35	22.31	22.65
17	21.91	21.06	21.63	22.04	22.80
Group Factor, $\bar{B}_{15}$					
7	20.00	20.22	20.07	20.62	20.11
9	20.11	21.14	19.95	20.12	20.14
11	20.57	21.72	20.32	19.98	20.58
13	21.65	23.03	22.10	21.30	21.09
15	22.13	22.97	22.17	21.99	21.65
17	22.12	22.81	22.33	22.21	21.81

enter positively into the equation, or through low mean values of the weight and cross-sectional and other measures which enter negatively into the equation, or through both. A study of the means of the sections in the original measurements showed, for these groups as for similar ones studied by Shuttleworth (9), that the later maturing girls do not attain superiority over the early maturing girls until the age of seventeen in the linear measures, but that they consistently lag behind in the cross-sectional measures from 11 to 17. The estimates of β are, therefore, not so much indicative of linear growth itself, as of a linear type of body, i.e., high mean estimates of β are characteristic of a group of girls who tend to have lighter weight and smaller cross-sectional measurements for the same height and limb lengths.

Interpreting β , then, as a measure of linearity of type, Figure 4 indicates that between the ages of 7 and 11 the girls of all four sections are stretching up, becoming more linear in type, and without significant differences between the sections. With puberty this tendency is checked, the filling out process becomes relatively more important and the mean estimates for the sections indicate a decrease in linearity. This is true for Section I after 11, these girls having reached puberty

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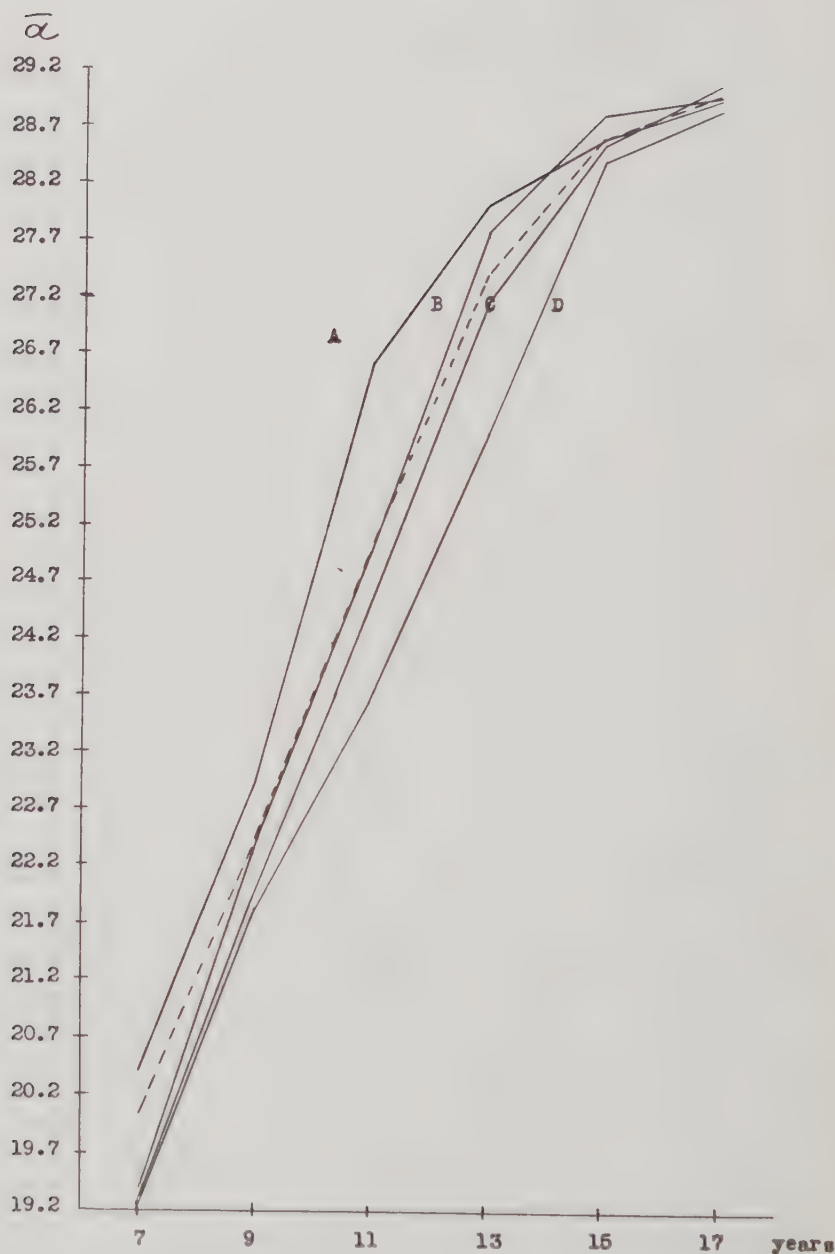


Fig. 3. Comparison of the mean growth of the general factor for four maturity sections and for all subjects (according to the equation for $\bar{\alpha}_{9b}$).

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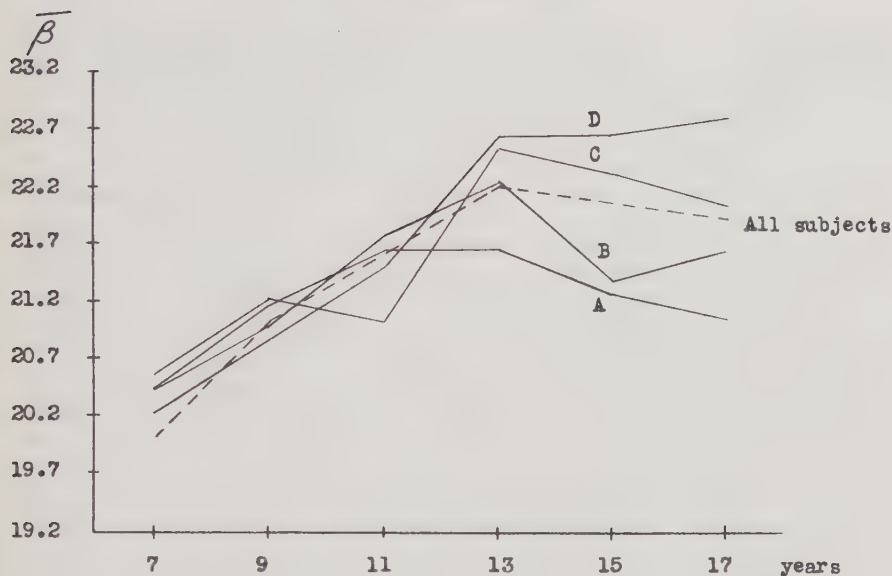


Fig. 4. Comparison of the mean growth of the group factor β for four maturity sections and for all subjects.

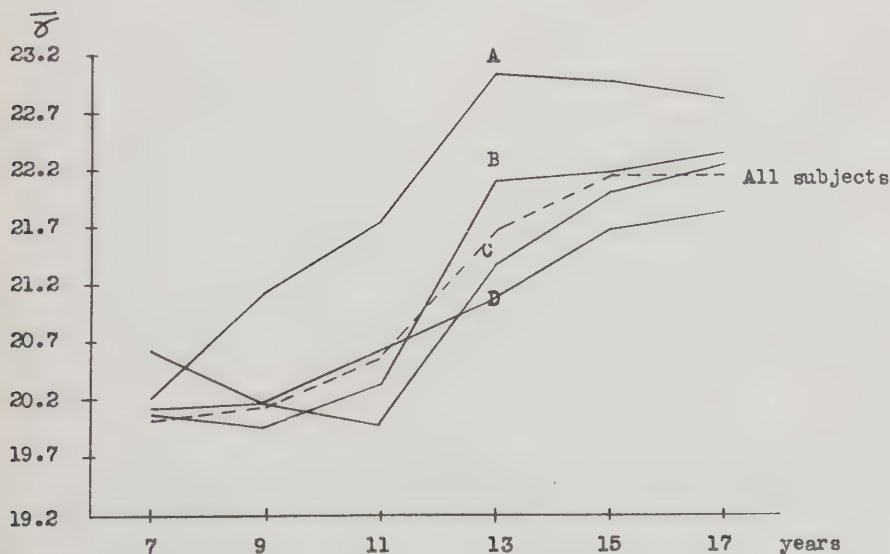


Fig. 5. Comparison of the growth of the group factor γ for four maturity sections and for all subjects.

at or before 11. For Sections II and III this is true at age 13, at which age or earlier these girls had attained puberty. Only for Section IV, the latest maturing group, does the tendency to linearity not only persist but increase up to the limits of this study.

The girls of the latest maturing section appear, therefore, to be on the average permanently of a more linear type than are the girls of the earlier maturing sections.

These deductions from the β curves are verified by the complementary picture shown in the σ curves. High estimates of σ would be produced either by high mean values of the cross-sectional measures which enter positively into the regression equation, or by low mean values of the height and limb and other measures which enter negatively into the equation, or by both conditions. The relative weightings of the different measures in the two regression equations are of course different and some measures enter negatively into both equations, so that the factor is by no means merely a measure of the obverse of β . The factors in fact are by hypothesis statistically independent. The factor σ is, however, a measure of the relationship of cross-sectional measures to height, rather than of cross-sectional measures alone.

The three sections II, III, and IV, all appear to follow a σ -growth curve similar to that for the whole group, showing little change from 7 to 11 with rapid increase in the prepubertal years and a retardation in the rate of increase thereafter. The girls who reach puberty earliest, however, show a very different trend of growth in this factor, with rapid increase in σ all the way from 7 to 13, during both prepubertal and one or more postpubertal years, and maintain a wide superiority over the other sections in respect to this factor, at least through the age of 17.

If the σ factor is interpreted as an expression of some underlying influence tending to accelerate growth in weight and the hip and chest measurements more rapidly than growth in the other dimensions, in other words of a tendency to stockiness of build, then the curves of Figure 5 may be said to indicate that the girls who will reach puberty at an early age show this tendency from the age of 7 on, while girls of the later maturing groups do not show this tendency till after 11. The girls of the earliest maturing section show persistently a much higher index in this trait than do the other sections, through the age of 17. For girls who will not mature so early, this factor does not come into operation till after 11. Thereafter it produces its effect most rapidly in the prepubertal years, the rate of change in body type produced by it being sharply decelerated at about the age of puberty.

SUMMARY

Factorial analyses have been made by the bi-factor technique of six tables showing the intercorrelations of seventeen physical measurements of girls of the ages respectively of seven, nine, eleven, thirteen, fifteen, and seventeen.

At all age levels studied a general size factor may be postulated which accounts for from thirty to fifty per cent of the total variance of the seventeen measurements.

The general factor as derived from the correlation matrices for the

various age groups is fairly consistent although at the earlier ages the linear measures correlate more highly with it and at the later ages the cross-sectional measures have higher loadings. The differences are not so great, however, but that the estimates of the general factor based on the six different factor patterns present very similar mean growth curves for the whole group, and that estimates made by one correlate highly with estimates made by another.

At all age levels studied, two group factors are apparent after the effect of the general factor is removed. One of these is found in the residual correlations of height, span of arms, length of forearm, and length of lower leg. Sitting height proved not to be significantly correlated with this factor. The proportion of the variance ascribable to this factor varies from 3.6 per cent at age 7 to 12.4 per cent at age 17. Estimates of this factor by regression equations calculated from different factor patterns do not correlate as highly with each other and do not give as similar growth curves as do the various estimates of the general factor.

The other group factor γ is found in the residual correlations for weight, bi-iliac and bi-trochanteric diameter, and chest girth, width, and depth. Shoulder width was the only cross-sectional trunk measure which did not have significant correlations with this factor at all ages, and it does have at ages 9 and 11. Factor loadings and regression weights for this factor also are not consistent from age to age. Its contribution to the total variance varies from 6 per cent for the seven-year-old group to 11.8 per cent at age 9.

The present data indicate that these two factors do exist at all ages, but it is probable that the measures used do not give sufficiently pure estimates of the group factors to make their prediction of much value at the present time.

Estimates of the general and group factors for girls of different maturity sections indicate that the general factor is closely related to the stage of sexual maturation. Growth curves for the estimates of the general factor show rapid growth in the years just preceding puberty and deceleration thereafter, with no permanent superiority for the early maturing group.

The group factors appear to be indicators of body type, the β factor giving crude measures of some underlying factor tending toward linearity of body type and the γ factor giving crude measures of some underlying factor tending toward stockiness of body type.

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APPENDIX

TABLE 11

CORRELATIONS OF SEVENTEEN PHYSICAL MEASUREMENTS OF
189 SEVEN-YEAR-OLD GIRLS^a

	Height	Arm Span	Length of Forearm	Length of Lower Leg	Sitting Height	Weight	Bi-iliac Diameter	Bi-trochanteric Diameter	Chest Girth	Chest Width	Chest Depth	Shoulder Width	Head Length	Head Width	Head Height	Hand Squeeze
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1																
2	867															
3	873	901														
4	904	869	856													
5	807	638	639	627												
6	726	678	653	735	662											
7	604	565	581	560	555	754										
8	652	596	565	635	597	838	744									
9	543	550	501	605	515	852	623	715								
10	455	466	427	499	393	634	616	629	571							
11	433	368	349	436	465	630	520	542	591	436						
12	575	605	524	560	514	643	587	667	544	614	330					
13	407	365	358	366	376	310	245	242	231	247	159	309				
14	276	259	240	243	288	275	320	321	167	322	160	299	155			
15	326	250	214	291	298	276	155	279	251	195	162	272	424	285		
16	288	268	278	270	268	296	214	187	256	160	197	208	077	068	015	
17 ^b	336	276	259	270	341	311	242	301	325	206	287	240	141	159	174	155

^aDecimal point properly preceding each entry has been omitted.

^bVariable 17 = lung capacity.

TABLE 12

CORRELATIONS OF SEVENTEEN PHYSICAL MEASUREMENTS OF
212 NINE-YEAR-OLD GIRLS^a

	1 Height	2 Arm Span	3 Length of Forearm	4 Length of Lower Leg	5 Sitting Height	6 Weight	7 Bi-iliac Diameter	8 Bi-trochanteric Diameter	9 Chest Girth	10 Chest Width	11 Chest Depth	12 Shoulder Width	13 Head Length	14 Head Width	15 Head Height	16 Hand Squeeze
1																
2	859															
3	872	914														
4	907	863	845													
5	807	652	699	656												
6	662	578	636	704	575											
7	564	549	575	572	503	770										
8	526	544	520	540	492	816	773									
9	489	524	489	569	396	856	639	707								
10	420	466	418	422	376	642	633	666	592							
11	397	394	399	431	390	720	572	608	713	494						
12	530	621	536	538	427	611	589	632	512	659	450					
13	396	367	366	425	360	360	303	264	236	262	245	287				
14	172	150	189	201	178	260	253	205	169	107	124	204	189			
15	277	237	200	296	245	256	184	201	258	167	102	176	425	255		
16	474	371	392	462	439	420	361	292	305	321	237	317	338	088	070	
17 ^b	427	449	443	402	437	299	275	263	214	315	214	338	260	116	144	434

^aDecimal point properly preceding each entry has been omitted.

^bVariable 17 = lung capacity.

TABLE 13

CORRELATIONS OF SEVENTEEN PHYSICAL MEASUREMENTS
OF 230 ELEVEN-YEAR-OLD GIRLS^a

Height	Arm Span	Length of Forearm	Length of Lower Leg	Sitting Height	Weight	Bi-iliac Diameter	Bi-trochanteric Diameter	Chest Girth	Chest Width	Chest Depth	Shoulder Width	Head Length	Head Width	Head Height	Hand Squeeze
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1															
2	899														
3	900	940													
4	916	900	887												
5	871	718	763	717											
6	678	673	667	728	612										
7	641	640	647	651	609	835									
8	673	686	692	698	613	874	857								
9	536	584	571	633	469	884	747	796							
10	534	572	548	576	472	752	768	734	750						
11	473	471	459	516	418	767	637	688	716	615					
12	580	679	639	615	495	640	642	667	596	715	474				
13	323	306	309	325	341	336	289	245	236	252	262	260			
14	250	217	202	250	206	265	291	253	242	239	147	269	158		
15	387	334	323	348	351	305	281	275	273	255	196	266	386	311	
16	555	536	553	486	548	455	410	432	393	315	321	330	315	097	196
17	529	511	493	466	536	413	380	430	362	358	327	419	250	091	204 437

^aDecimal point properly preceding each entry has been omitted.

^bVariable 17 = lung capacity.

TABLE 14

CORRELATIONS OF SEVENTEEN PHYSICAL MEASUREMENTS
OF 219 THIRTEEN-YEAR-OLD GIRLS^a

	Height	Arm Span	Length of Forearm	Length of Lower Leg	Sitting Height	Weight	Bi-iliac Diameter	Bi-trochanteric Diameter	Chest Girth	Chest Width	Chest Depth	Shoulder Width	Head Length	Head Width	Head Height	Hand Squeeze
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1																
2	872															
3	859	905														
4	869	815	827													
5	813	626	638	528												
6	583	526	491	522	603											
7	544	437	451	432	562	724										
8	637	539	529	543	615	822	735									
9	336	369	297	363	310	772	498	601								
10	417	437	367	401	367	696	584	596	625							
11	276	272	267	301	277	604	411	470	555	438						
12	527	572	489	526	411	597	500	596	461	699	332					
13	338	353	320	253	346	314	244	252	189	176	253	277				
14	227	165	149	213	253	236	252	271	136	171	084	281	225			
15	397	287	271	333	406	348	318	376	229	361	180	385	292	472		
16	337	349	329	289	357	430	357	344	338	361	307	285	351	112	243	
17 ^b	539	537	487	457	442	422	358	397	331	422	251	454	231	120	185	431

^aDecimal point properly preceding each entry has been omitted.

^bVariable 17 = lung capacity.

TABLE 15

CORRELATIONS OF SEVENTEEN PHYSICAL MEASUREMENTS
OF 305 FIFTEEN-YEAR-OLD GIRLS^a

	Height	Arm Span	Length of Forearm	Length of Lower Leg	Sitting Height	Weight	Bi-iliac Diameter	Bi-trochanteric Diameter	Chest Girth	Chest Width	Chest Depth	Shoulder Width	Head Length	Head Width	Head Height	Hand Squeeze
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1																
2	846															
3	805	881														
4	859	826	801													
5	740	497	494	451												
6	473	376	380	436	507											
7	457	349	361	332	502	630										
8	398	326	319	329	445	762	662									
9	301	277	237	327	327	730	404	583								
10	382	415	345	365	305	629	487	577	539							
11	201	148	194	227	211	611	319	415	484	368						
12	426	546	426	388	327	406	285	424	347	599	139					
13	276	192	236	200	321	244	135	190	117	112	188	149				
14	302	218	168	249	272	319	204	294	320	265	127	290	107			
15	251	179	137	213	250	142	135	190	183	218	179	128	283	277		
16	291	307	344	258	287	418	281	336	354	321	224	300	158	123	051	
17 ^b	484	485	428	409	415	419	344	398	427	435	352	361	214	178	240	413

^aDecimal point properly preceding each entry has been omitted.

^bVariable 17 = lung capacity.

TABLE 16

CORRELATIONS OF SEVENTEEN PHYSICAL MEASUREMENTS
OF 165 SEVENTEEN-YEAR-OLD GIRLS^a

	Height	Arm Span	Length of Forearm	Length of Lower Leg	Sitting Height	Weight	Bi-iliac Diameter	Bi-trochanteric Diameter	Chest Girth	Chest Width	Chest Depth	Shoulder Width	Head Length	Head Width	Head Height	Hand Squeeze
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1																
2	790															
3	788	898														
4	833	836	832													
5	677	422	389	421												
6	502	437	406	446	383											
7	387	263	189	279	288	621										
8	411	341	295	323	343	710	579									
9	314	353	293	407	253	664	342	503								
10	365	387	295	292	269	696	534	591	507							
11	140	111	124	191	129	444	224	289	373	309						
12	409	568	432	457	305	581	352	553	453	679	282					
13	296	198	196	242	341	372	180	338	233	260	126	216				
14	139	134	062	128	101	104	023	066	202	174	021	227	052			
15	158	045	084	112	277	197	228	233	117	183	213	168	361	104		
16	154	178	187	164	149	350	152	223	221	313	180	207	116	053	058	
17 ^b	424	456	416	408	335	364	231	258	387	433	277	476	156	182	120	253

^aDecimal point properly preceding each entry has been omitted.

^bVariable 17 = lung capacity.

